On the possibility to observe higher n^3D_1 bottomonium states in the e^+e^- processes

A.M. Badalian*
Institute of Theoretical and Experimental Physics, Moscow, Russia

B.L.G. Bakker†
Department of Physics and Astronomy, Vrije Universiteit, Amsterdam, The Netherlands

I.V. Danilkin‡
Moscow Engineering Physics Institute, Moscow, Russia and
Institute of Theoretical and Experimental Physics, Moscow, Russia

The possibility to observe new bottomonium states with $J^{PC} = 1^{--}$ in the region $10.7 - 11.1$ GeV is discussed. The analysis of the di-electron widths shows that the $(n+1)\,^3S_1$ and $n\,^3D_1$ states $(n \geq 3)$ may be mixed with a rather large mixing angle, $\theta \approx 30^\circ$ and this effect provides the correct values of $\Gamma_{ee}(\Upsilon(10580))$ and $\Gamma_{ee}(\Upsilon(11020))$. On the other hand, the $S-D$ mixing gives rise to an increase by two orders of magnitude of the di-electron widths of the mixed $\tilde{\Upsilon}(n\,^3D_1)$ resonances $(n = 3, 4, 5)$, which originate from pure $D-$wave states. The value $\Gamma_{ee}(\tilde{\Upsilon}(3D)) = 0.095^{+0.028}_{-0.025}$ keV is obtained, being only ~ 3 times smaller than the di-electron width of $\Upsilon(10580)$, while $\Gamma_{ee}(\tilde{\Upsilon}(5D)) \sim$ 135 eV appears to be close to $\Gamma_{ee}(\Upsilon(11020))$ and therefore this resonance may become manifest in the e^+e^- experiments. The mass differences between $M(nD)$ and $M((n+1)S)$ $(n = 4, 5)$ are shown to be rather small, 50 ± 10 MeV.

I. INTRODUCTION

Recently the Belle Collaboration has observed an enhancement in the production process, $e^+e^- \to \Upsilon(nS)\pi^+\pi^-$ $(n = 1, 2, 3)$ [1]. Their fit using a single Breit-Wigner resonance yields a resonance mass 10889.6(3.3) MeV, slightly larger than that of $\Upsilon(10860)$, and a width $54.7^{+11.0}_{-9.9}$ MeV, which is two times smaller than the width of $\Upsilon(10860)$, known from the earlier experiments [2], [3]. The BaBar Collaboration has also observed two resonance structures in the $e^+e^- \to b\bar{b}$ cross sections between 10.54 and 11.20 GeV with the fitting parameters: $M_5 = 10876(2)$ MeV, $\Gamma_5 = 43(4)$ MeV and $M_6 = 10996(2)$ MeV, $\Gamma_6 = 37(3)$ MeV [4], which also differ from the parameters of the conventional $\Upsilon(10865)$ and $\Upsilon(11020)$ resonances.

Meanwhile, precise knowledge of the masses and the di-electron widths of higher bottomonium vector states is very important for the theory: They may provide new information on the details of the QCD quark-antiquark interaction at large distances, possible hadronic shifts of higher states, like $\Upsilon(10860)$ and $\Upsilon(11020)$, and $S-D$ mixing. At present it remains unclear whether it is possible to observe the higher $n\,^3D_1$ $(n = 3, 4, 5)$ states, which have masses in the mass region considered [5]-[7].

It is known that pure $D-$wave bottomonium states have very small di-electron widths [7], [8], in particular, in Ref. [7] the values $\Gamma_{ee}(n,\,^3D_1) \sim 1-2$ eV are obtained. Therefore an observation of the $D-$wave resonances in the e^+e^- processes seems to be not possible now. However, one cannot exclude that the bottomonium D-wave states with $J^{PC} = 1^{--}$, which lie above the open beauty threshold(s), may be mixed with the nearby $S-$wave states, as it takes place in the charmonium family, where due to $S-D$ mixing the di-electron widths of physical resonances, e.g. $\psi(4040)$ and $\psi(4160)$, have almost equal di-electron widths [9].

An important feature of the bottomonium spectrum is that the mass difference between the $(n+1)S$ and nD states is small and decreases for increasing n. In [7] the value $\Delta M(n) \sim 50(10)$ MeV for $n \geq 3$ was obtained, if the coupling to open channel(s) is not taken into account, although the coupling to the $B\bar{B}$ and $B_s\bar{B}_s$ channels may be strong [10]. Owing to such a coupling a mass shift of the higher resonances may occur. In particular, the mass shift down of $\Upsilon(4S)$ is estimated to be ~ 50 MeV.

Up to now only the $1D-$meson with $J^{PC} = 2^{--}$ and $M(1D) = 10161(2)$ MeV has been measured by the CLEO Collaboration in the cascade radiative processes [11], which lies far below the $B\bar{B}$ threshold. Here we will discuss mostly those bottomonium states which are above the $B\bar{B}$ theshold, and concentrate on those resonances which originate from pure $D-$wave states $(n \geq 3)$. Observation of such "$D-$wave" resonances in the e^+e^- processes may be possible, if owing to $S-D$ mixing their di-electron widths are not small.

At present the resonances $\Upsilon(10580)$, $\Upsilon(10860)$, and $\Upsilon(11020)$ are usually considered as pure $n\,^3S_1$ $(n = 4, 5, 6)$ states. However, in theoretical studies with different $Q\bar{Q}$ potentials [6], [7] their di-electron widths turn out to be significantly larger than those found in experiment. We do not support the point of view of the au-

*Electronic address: badalian@itep.ru
†Electronic address: blg.bakker@few.vu.nl
‡Electronic address: danilkin@itep.ru

thors of Ref. [5] who, in order to suppress the calculated di-electron widths, took a small QCD radiative correction factor $\beta_V = 0.46$ (our notation), which corresponds to very large value of $\alpha_s(\sim 2m_b) = 0.317$ and therefore decreases the di-electron widths by a factor of two. Moreover, in [5] and [6] the $S-D$ mixing is not taken into account.

A detailed study of the di-electron widths for all nS and nD ($n = 1, ...6$) vector states in [7] shows that the calculated widths are $\sim 25\%$ larger for $4S$ and two times larger for $\Upsilon(11020)$, while for all other states the di-electron widths agree with experiment with high accuracy, better 3% [7]. These facts can be considered as an indirect indication of a possible $S-D$ mixing between higher vector states in bottomonium and our letter is just devoted to this topic.

II. COMPARISON OF CALCULATED RESULTS TO DATA

The study of the bottomonium spectrum done here and in [7], uses the single-channel relativistic string Hamiltonian (RSH) with a universal potential [12]. This Hamiltonian has been derived from the gauge-invariant meson Green's function in QCD and in bottomonium it has an especially simple form:

$$H_0 = \omega + \frac{\mathbf{p}^2 + m_b^2}{\omega} + V_B(r). \tag{1}$$

In general, the quantity ω appearing in this expression is a n operator, which has to defined by an extremum condition, exiting in two forms: If the extremum condition is put on H_0, then one obtains the well-known spinless Salpeter equation (SSE), thus establishing a direct connection between the SSE and the QCD meson Green's function. In the second case the extremum condition is put on the eigenvalue, or the meson mass, which give rise to the Einbein approximation (EA) [9]. We use here the EA because it has an important advantage as compared to the SSE: Its S-wave functions are finite at the origin, while they diverge near the origin in the SSE and need to be regularized, adding a number of additional unknown parameters.

The potential $V_B(r)$ in (1) is the sum of a pure scalar confining term and a gluon-exchange part,

$$V_B(r) = \sigma\, r - \frac{4}{3}\frac{\alpha_B(r)}{r}, \tag{2}$$

where the vector coupling $\alpha_B(r)$ is taken in two-loop approximation and possesses two important features: the asymptotic freedom behavior at small distances, defined by the QCD constant $\Lambda_B(n_f)$ [which is considered to be known, because Λ_B is directly expressed via the QCD constant $\Lambda_{\overline{MS}}(n_f)$ in the $\overline{MS}$ renormalization scheme]; it freezes at large distances. Details about the effective fine-structure constant can be found in Ref. [9].

TABLE I: Spin-averaged masses in MeV/c^2 of the higher nD and $(n+1)S$ states in the region $10.4 - 11.1$ GeV.

n	1	2	3	4	5
$M(nD)$	10 140	10 440	10 700	10 920	11 115
$M((n+1)S)$	10 015	10 360	10 640	10 870	11 075

The RSH has been successfully applied to light mesons [13], heavy-light mesons [14], and heavy quarkonia [15]. Within this approach relativistic corrections are taken into account and a higher state can be considered on the same grounds as a lower one; still at present the coupling to open channel(s) is neglected. Nevertheless, for higher states the calculated masses appear to be rather close to the experimental ones and we can estimate possible mass shifts due to a coupling to open channel(s): A comparison does not give large shifts, $\sim 50 \pm 10$ MeV for $\Upsilon(10580)$ and $\Upsilon(11020)$. Still it remains unclear why for $\Upsilon(10860)$ the calculated and experimental masses coincide. It seems possible that no hadronic shift occurs in this case.

For our analysis it is of great importance that another effect, namely, the production of virtual light quark pairs, is taken into account. This effect gives rise to a flattening of the confining potential [16] and due to this flattening phenomenon correlated downward shifts of the masses of the higher states occur, in particular, the shift of the $6S$-state is ~ 40 MeV.

The spectrum and di-electron widths of higher bottomonium states have several characteristic features.

1. In the numbers given in Table I the theoretical error ± 15 MeV is not included; it mostly comes from an uncertainty in our knowledge of the pole (current) b-quark mass, taken here equal to $m_b(\text{pole}) = 4.825$ GeV.

   As shown in Table I, the masses of the nD states ($n = 3, 4, 5$) occur just in the mass region $10.7 - 11.1$ GeV, which has been studied in the experiments [1], [4]. Still, one cannot exclude that due to the coupling to open channel(s) the physical masses of the mixed nD states may slightly differ, as is the case for $\Upsilon(10580)$ and $\Upsilon(11020)$.

2. The mass difference between the n^3D_1 and $(n+1)^3S_1$ states

$$\Delta_n = M(nD) - M((n+1)S), \tag{3}$$

   decreases for growing n: from ~ 140 MeV for $n = 1$ (from experiment), ~ 60 MeV for $n = 3$ up to the small value ~ 40 MeV for $n = 5$. Due to such a small difference the probability of the $S-D$ mixing between higher bottomonium vector states increases.

3. While the $n\,{}^3D_1$ state (for a given $n \geq 3$) is mixed with the $(n+1)\,{}^3S_1$ state, such a mixed "$D-$wave"

state, denoted below as $\tilde{\Upsilon}(nD)$, will have a significantly larger di-electron width than a pure $D-$wave state, even if the mixing angle is not large.

In the case of charmonium, the almost equal di-electron widths of $\psi(4160)$ and $\psi(4040)$, also found in experiment, have been obtained only for a large mixing angle, namely, $\theta \cong 35^\circ$ [9]. For $\psi(3686)$ and $\psi(3770)$ the mixing angle, $\theta \cong 10^\circ$, is significantly smaller [17], [18]; nevertheless, the experimental value $\Gamma_{ee}(3770) = 0.247$ keV appears to be ~ 10 times larger than that of a pure $1\,^3D_1$ state.

4. The di-electron widths of pure $n\,^3D_1$ bottomonium states are very small, $\sim (1-2)$ eV. They are denoted below as $\Gamma^0_{ee}(nD)$, and given in Table II

TABLE II: The di-electron widths (in keV) of pure $(n+1)\,^3S_1$ and $n\,^3D_1$ states in bottomonium from [7] and experimental numbers from [3].

n	1	2	3	4	5
$\Gamma^0_{ee}(nD)$	0.62×10^{-3}	1.08×10^{-3}	$1.44\cdot10^{-3}$	1.71×10^{-3}	1.9×10^{-3}
$\Gamma^0_{ee}((n+1)S)$	0.614	0.448	0.37	0.316	0.274
$\Gamma_{\exp}(\Upsilon((n+1)S))$	0.612(11)	0.443(8)	0.272(29)	0.31(7)	0.13(3)
$\frac{\Gamma^0_{ee}(nD)}{\Gamma^0_{ee}((n+1)S)}\times10^3$	1.0	2.4	3.9	5.4	6.9

For the ground state $\Upsilon(9460)$ we have obtained $\Gamma_{ee}(\Upsilon(9460)) = 1.317$ keV, in great agreement with the experimental number, equal to 1.34 ± 0.02 keV. Also, as seen from Table II, the values $\Gamma_{ee}(nS)$ $(n=2,3)$ coincide with precise accuracy with the experimental widths of $\Upsilon(10023)$ and $\Upsilon(10355)$. For the low-lying states the ratios $r(m/n) = \Gamma_{ee}(mS)/\Gamma_{ee}(nS)$ of the calculated widths ($\Gamma_{ee}(1S) = 1.317$ keV, $\Gamma_{ee}(2S) = 0.614$ keV, and $\Gamma_{ee}(3S) = 0.448$ keV) are found to be $r(2/1) = 0.466$, $r(3/1) = 0.340$, and $r(3/2) = 0.730$, which agree with the experimental numbers from [19]: $r_{\exp}(2/1) = 0.457(8)$, $r_{\exp}(3/1) = 0.329(6)$, and $r_{\exp}(3/2) = 0.720(16)$ with an accuracy better than 3%.

For a better understanding of the e^+e^- dynamics it is important that in our analysis the same QCD radiative correction factor, $\beta_V = 1 - \frac{16}{3\pi}\ \alpha_s(2m_b)$ is taken. This factor is cancelled in the ratios of the di-electronic widths and this result indicates that the calculated values of the wave function (w. f.) at the origin are defined with a good accuracy. Then β_V can be extracted from the absolute values of $\Gamma^0_{ee}(nS)$ $(n\le3)$, giving the same $\beta_V = 0.80$ for all low-lying states. This value of β_V shows that in bottomonium the one-loop QCD corrections decrease the di-electron widths by only 20% (while in [5] $\beta_V \simeq 0.5$, being even smaller than in the charmonium family, where $\beta_V \simeq 0.62(2)$ is used in [9]).

However, for the states above the $B\bar{B}$ threshold we obtain widths which are two times larger for the $6S$ state and $\sim 25\%$ larger for the $4S$ vector state. The reasons behind such a suppression of the di-electron widths for higher states has been discussed in [6], where, however, the $S-D$ mixing is not taken into account. In particular, there it has been demonstrated that the di-electron widths, calculated in the framework of the Cornell coupled-channel model [20], are not suppressed. Moreover, we expect that an open channel cannot essentially modify the w.f. at the origin, because, as shown in [21], the w.f. at the origin of a four-quark system (like $Q\bar{Q}q\bar{q}$) is much smaller than that of a meson ($Q\bar{Q}$). It means that a continuum channel, considered as a particular case of a four-quark system, cannot significantly affect the meson w.f. at the origin. Therefore we assume here that in bottomonium, as well as in the charmonium family, the w.f. at the origin, and as a consequence the di-electron widths, decrease mostly due to the $S-D$ mixing.

To get into agreement with the experimental value $\Gamma_{ee}(\Upsilon(10580)) = 0.272(29)$ keV, we take into account the $4S-3D$ mixing with the fitting angle, $\theta = (27\pm5)^\circ$, which appears to be not small (see Table III).

Surprisingly, for the $5S$ state the calculated width coincides with the experimental central value, $\Gamma_{ee}(\Upsilon(10860)) = 0.31(7)$ [3]. Since for $\Upsilon(10860)$ the width has a large experimental error, $\le 20\%$, one cannot conclude whether $5S-4D$ mixing takes place or not. To answer this question, more precise measurements of $\Gamma_{ee}(10860)$ are needed. For an illustration we give in Table III the width for the mixing angle $\theta = 27^\circ$. Its value $\Gamma_{ee}(\Upsilon(10860)) = 0.23$ keV coincides with the lower bound of the experimental number.

For $\Upsilon(11020)$ its di-electron width, $\Gamma_{ee}(11020) = (0.13\pm3)$ keV is two times smaller than the calculated number for $\theta=0$ and by 26% smaller than for $\theta = 27^\circ$. To obtain such a small width we have taken a larger mixing angle for $\Upsilon(11020)$, considereing this resonance not as a pure $6\,^3S_1$ state. Good agreement with experiment is obtained for the mixing angle $(40\pm5)^\circ$, for which almost the same number occurs for $\tilde{\Upsilon}(5D)$, the mixed $5D$

TABLE III: The di-electron widths of the $(n+1)\,^3S_1$ and $n\,^3D_1$ states (in keV) without mixing ($\theta = 0$) and with $S-D$ mixing ($\theta = 27^\circ$). The experimental numbers are taken from [3].

	Theory		Experiment
	$\theta = 0$	$\theta = 27^\circ$	
$\Gamma_{ee}(4S)$	0.37	0.275	0.272±0.029
$\Gamma_{ee}(3D)$	1.44×10^{-3}	0.095	Absent
$\Gamma_{ee}(5S)$	0.316	0.232	0.31± 0.07
$\Gamma_{ee}(4D)$	1.715×10^{-3}	0.085	Absent
$\Gamma_{ee}(6S)$	0.274	0.199	0.13± 0.03
$\Gamma_{ee}(5D)$	1.9×10^{-3}	0.076	Absent

state, namely

$$\begin{cases} \Gamma_{ee}(\Upsilon(11020)) & = & 0.139(25) \;\; \text{keV} \\ \Gamma_{ee}(\tilde{\Upsilon}(5D)) & = & 0.136(25) \;\; \text{keV}. \end{cases} \quad (4)$$

It is of interest to notice that close value of the mixing angle $\theta \cong 35^\circ$ has been extracted in [13] to obtain the di-electron widths of $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$ in agreement with experiment.

III. SUMMARY AND CONCLUSION

Our study of higher $D-$wave states shows that their masses are close to those of the $(n+1)S$ resonances and their di-electron widths are not small, $\geq$ 70 eV, if the $S-D$ mixing is taken into account. There are three arguments in favor of such a mixing:

1. Suppression of the di-electron widths of $\Upsilon(10580)$ and $\Upsilon(11020)$.
2. Strong coupling to the $B\bar{B}$ $(B_s\bar{B}_s)$ channel, which has become manifest in the recent observations of the resonances in the processes like $e^+e^- \to \Upsilon(nS)\pi^+\pi^-$ $(n = 1, 2, 3)$ [1] and supported by the theoretical analysis in [10].
3. Similarity with the $S-D$ mixing in the charmonium family.

The important question arises whether it is possible to observe mixed $D-$wave states in e^+e^- experiments. Our calculations give $M(3D) \sim 10700$ MeV (not including a possible hadronic shift) and $\Gamma_{ee}(\tilde{\Upsilon}(3D)) \sim 95$ eV, which is three times smaller than $\Gamma_{ee}(\Upsilon(10580))$. For such a width an enhancement from this resonance in the e^+e^- processes will be suppressed, as compared to the peak of the $\Upsilon(10580)$ resonance.

The di-electron width of $\Upsilon(10860)$ contains a rather large experimental error and therefore one cannot draw a definite conclusion concerning the possibility of $5S-4D$ mixing, while for the $4D$ state the mass 10920 ± 15(th) MeV is obtained.

It is more probable to observe the resonance $\tilde{\Upsilon}(5D)$ (with the mass 11115 ± 15(th) MeV), for which the di-electron width can even be equal to that of the conventional $\Upsilon(11020)$ resonance. However, since the cross sections of different e^+e^- processes depend also on other unknown parameters, like the total width and branching ratio to hadronic channels, the possibility to observe a mixed $5D$-wave state, even for equal di-electron widths, might be smaller than for $\Upsilon(11020)$. In [4] only the $\Upsilon(11020)$ resonance has been observed in the mass region around 11 GeV. Still one cannot exclude that due to an overlap with an unobserved $\tilde{\Upsilon}(5D)$ resonance, the shape and other resonance parameters of the conventional $\Upsilon(11020)$ resonance can be distorted.

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